ANALYTICAL METHOD FOR DETERMINING ERRORS IN CURRENT MEASUREMENTS WITH A ROGOWSKI COIL

M. J. Hagmann, NewPath Research L.L.C., P. O. Box 3863, Salt Lake City, UT 84110, U.S.A.

Abstract

Algebraic expressions are derived for the open-circuit voltage induced on a toroidal coil with unevenly spaced turns, by a sinusoidal current through its aperture. The derivation requires that the winding layer is thin, the number of turns is large, and the cross-section of the winding is rectangular. These expressions are used to determine the effects of the gap between the ends of the coil and other irregularities in the spacing of the turns on the position sensitivity, defined as the dependence of the induced voltage on the location of the current. This technique may be used to define the criteria to meet a specified upper limit for the positional sensitivity.

INTRODUCTION

A Rogowski Coil [1] is a non-ferrous current probe in which a uniformly wound helix with a constant cross-sectional area follows a closed curve of arbitrary shape. Faraday's law of induction, with Ampere's law [2], requires that the open circuit voltage V induced on the coil is given by

$$V = \mu_0 N' A \frac{dI}{dt} \tag{1}$$

where N' is the number of turns per unit length of the helix, A is the cross-sectional area of the helix, and dI/dt is the rate of change of the current passing through the area enclosed by the closed curve.

An ideal Rogowski coil would provide equal sensitivity to currents that are located anywhere within the aperture and would not be sensitive to currents outside of the aperture. However, the derivation of Eq. (1) requires that (1) The winding is uniform; (2) The radius of the winding is much less than the distance from the current to the coil; (3) N' is large so the helix may be approximated by a large number of evenly-spaced coils that are each normal to the curved axis of the helix; and (4) The frequency is low enough that displacement current and transit time may be neglected.

Measurements show that Rogowski coils have "position sensitivity" in that the induced voltage depends on the location of the current within the aperture [3]. Conventional methods of winding coils generally cause a positional sensitivity greater than 1%. A position sensitivity of less than 0.1% has been obtained by machining coils to a precision of 25-50 μm and reducing the gap where the two ends of the coil are adjacent to each other.

In the following sections of this paper we examine the effects of unevenly spaced turns in a toroidal Rogowski coil on the induced voltage when the cross-section of the winding is uniform and rectangular with arbitrary size. However, it is necessary to assume that the winding has zero thickness, so the present analysis does not show changes in the sensitivity when the distance of the current to the coil is comparable to the thickness of the winding. The number of turns per unit length is assumed to be large enough that the winding may be approximated by a group of closely-spaced coils in planes having constant θ (azimuthal coordinate about the axis through the center of the aperture). Otherwise the incremental advancement of each turn would create an undesirable one-turn loop to cause the coil to be sensitive to magnetic fields parallel to the axis. However, the latter effect may be mitigated by adding a one-turn return loop [4] or other methods which are not considered here. The present analysis is limited to frequencies that are low enough so the current probe is much smaller than a wavelength.

Our objective is to provide a tool to determine limits for the nonuniformity in the winding to satisfy a specified upper limit for the positional sensitivity. Others have used numerical integration to model these effects [5], but we have derived algebraic expressions that are intended to be simpler to implement and provide greater understanding.

ANALYSIS

Figure 1 shows the configuration used for the analysis of induction with a non-ferrous toroidal coil that may have a nonuniform winding. We consider the induction in an incremental winding of length $R_1 d\theta$, centered at (R_1,θ) that is caused by current I intersecting the plane at point $P(R_2,\Phi)$. Lines L_1 and L_2 are normal to the plane of the incremental winding, and parallel to the magnetic field caused by the current, respectively, and the angle between these two lines is α.

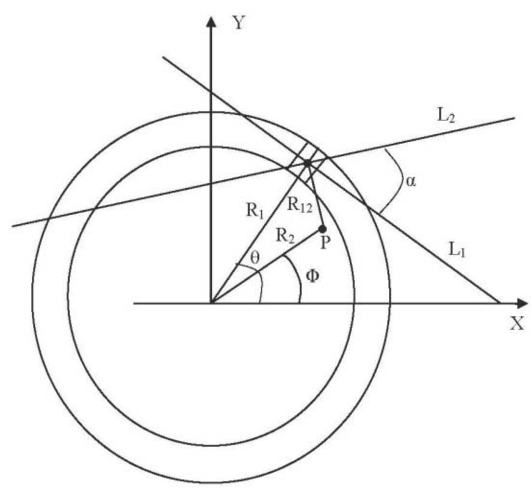

Fig. 1. Diagram for analysis.

The increment of winding has $dN = N'(\theta)R_1 d\theta$ turns, where $N'(\theta)$ is the turn density, defined as the number of turns per unit length of the toroid. Thus, the open-circuit voltage induced on the coil is given by

$$V_{OC} = \int_0^h dz \int_{r_1}^{r_2} dr \int_0^{2\pi} \frac{j\omega\mu_0 R_1 I_0 \cos(\alpha) N'(\theta) d\theta}{2\pi\sqrt{R_2^2 + r^2 - 2R_2 r\cos(\theta - \Phi)}} \quad (2)$$

The limits for integration over the cross-section of the winding are 0, h, r_1, and r_2; r is a dummy variable in place of R_1, and the cosine of α is given by

$$\cos(\alpha) = \frac{r - R_2\cos(\theta - \Phi)}{\sqrt{R_2^2 + r^2 - 2R_2 r\cos(\theta - \Phi)}} \quad (3)$$

Thus, combining Eqs. (2) and (3), the open-circuit voltage is given by

$$V_{OC} = \frac{j\omega\mu_0 R_1 I_0}{2\pi}\int_0^h dz \int_{r_1}^{r_2}\frac{dr}{r}\int_0^{2\pi}\frac{\left[r^2 - R_2 r\cos(\theta-\Phi)\right]N'(\theta)d\theta}{\left[R_2^2 + r^2 - 2R_2 r\cos(\theta-\Phi)\right]} \quad (4)$$

The turn density is given by the following Fourier series:

$$N'(\theta) = N_0' + \sum_{J=1}^{\infty} N_{JC}'\cos(J\theta) + N_{JS}'\sin(J\theta) \quad (5)$$

Substituting Eq. (5) into Eq. (4), and evaluating the integrals [6] gives the following expression for the open-circuit voltage on the Rogowski coil:

$$\begin{aligned} V_{OC} &= j\omega\mu_0 h \ln\left(\frac{r_2}{r_1}\right) R_1 N_0' I_0 \\ &+ \frac{j\omega\mu_0 h R_1 I_0}{2}\sum_{J=1}^{\infty}\frac{N_{JC}'\cos(J\Phi)}{J}\left[\left(\frac{R_2}{r_1}\right)^J - \left(\frac{R_2}{r_2}\right)^J\right] \\ &+ \frac{j\omega\mu_0 h R_1 I_0}{2}\sum_{J=1}^{\infty}\frac{N_{JS}'\sin(J\Phi)}{J}\left[\left(\frac{R_2}{r_1}\right)^J - \left(\frac{R_2}{r_2}\right)^J\right] \end{aligned} \quad (6)$$

For the special case of a thin toroid, so that $r_1 = R_1 - \Delta/2$ and $r_2 = R_1 + \Delta/2$ where $\Delta << R_1$, Eq. (6) may be simplified to give the following expression:

$$V_{OC} \approx j\omega\mu_0 h\Delta N_0' I_0 + \frac{j\omega\mu_0 h\Delta I_0}{2}\sum_{J=1}^{\infty}\left[\begin{array}{l} N_{JC}'\left(\frac{R_2}{R_1}\right)^J\cos(J\Phi) \\ +N_{JS}'\left(\frac{R_2}{R_1}\right)^J\sin(J\Phi)\end{array}\right] \quad (7)$$

When given a specified turn density N'(θ), the coefficients in the Fourier series of Eq. (5) may be determined using the following equations:

$$N_0' = \frac{1}{2\pi}\int_0^{2\pi} N'(\theta)d\theta \quad (8A)$$

$$N_{JC}' = \frac{1}{\pi}\int_0^{2\pi} N'(\theta)\cos(J\theta)d\theta \quad (8B)$$

$$N_{JS}' = \frac{1}{\pi}\int_0^{2\pi} N'(\theta)\sin(J\theta)d\theta \quad (8C)$$

EXAMPLES

1. Effect of a gap in the winding

Consider a coil having a gap over a fraction f of the total circumference, such as between the two adjacent ends of the coil. The gap is located from $-\pi f < \Phi < \pi f$ in a coil which elsewhere has a uniform turn density N'(θ) = N_0'. The Fourier coefficients are determined with Eqs. (8A)-(8C), and then Eq. (6) gives the following expression for the fractional error in the open-circuit voltage:

$$\frac{\Delta V_{OC}}{V_{OC0}} = -f - \frac{1}{\pi\ln(\gamma)}\sum_{J=1}^{\infty}\frac{\sin(fJ\pi)}{J^2}\left(\frac{R_2}{r_1}\right)^J\left(1 - \frac{1}{\gamma^J}\right)\cos(J\Phi) \quad (9)$$

where V_{OC0} is the value for the case where the width of the gap is zero and the relative width of the coil, $\gamma = r_2/r_1 > 1$. Equation (9) shows that the position sensitivity which is caused by a gap can always be reduced by increasing γ, as others have noted from their specific numerical solutions [5]. For the case of a thin toroid, Eq. (9) may be simplified to give the following expression:

$$\frac{\Delta V_{OC}}{V_{OC0}} = -f - \frac{1}{\pi}\sum_{J=1}^{\infty}\frac{\sin(fJ\pi)}{J}\left(\frac{R_2}{R_1}\right)^J\cos(J\Phi) \quad (10)$$

Figures 2 and 3 show the fractional error in the open-circuit voltage as a function of the angle Φ for a coil with a gap having a width of 1° (f = 1/360), determined with Eq. (14). These two figures also show the effect of the radial location of the current, with R_2/r_1 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 8.8, and 0.9 from the top curve to the lowest curve, respectively. Figures 2 (γ = 1.25) and 3 (γ = 3.0) show that the position sensitivity increases sharply as the current is moved closer to the gap, both azimuthally and radially, and it may be reduced by increasing the thickness of the coil.

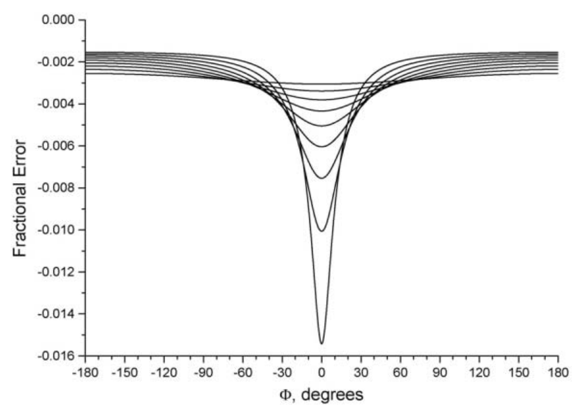

Fig. 2. Fractional error vs. the angle Φ for a coil with γ = 1.25, having a gap having a width of 1°.

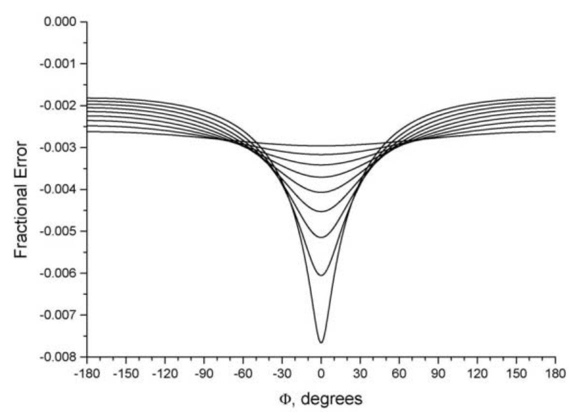

Fig. 3. Fractional error vs. the angle Φ for a coil with γ = 3.0, having a gap having a width of 1°.

2. Effect of a sinusoidal perturbation in winding the coil

When a single sinusoidal term from the Fourier series in Eq. (5) is added as a perturbation to a uniformly-wound coil, Eq. (6) shows that the peak value of the fractional error in the open-circuit voltage is given by

$$\frac{\Delta V_{OC}}{V_{OC0}} = F\left(J, \gamma, \frac{R_2}{r_1}\right)\frac{N_{JC}'}{N_0'} \qquad (11A)$$

$$\text{where } F\left(J, \gamma, \frac{R_2}{r_1}\right) \equiv \frac{1}{2J\ln(\gamma)}\left(\frac{R_2}{r_1}\right)^J\left(1-\frac{1}{\gamma^J}\right) \qquad (11B)$$

Figure 4 shows the value of the coefficient F(J, γ, R_2/r_1) from Eq. (11B) as a function of the normalized radial coordinate of the current (R_2/r_1) for a coil with $\gamma = 1.25$, having a sinusoidal perturbation with J = 1, 2, 3, 4, 5, 6, 8, and 10, from the top to the lowest curve, respectively. The fractional error in the open-circuit voltage is proportional to the Jth power of R_2/r_1, so terms with higher spatial frequency would have a small effect unless the current is close to the surface of the coil. As seen in the first example for a gap in the winding, the error that is caused by a single sinusoidal term from the Fourier series may also be reduced by increasing the relative width of the coil.

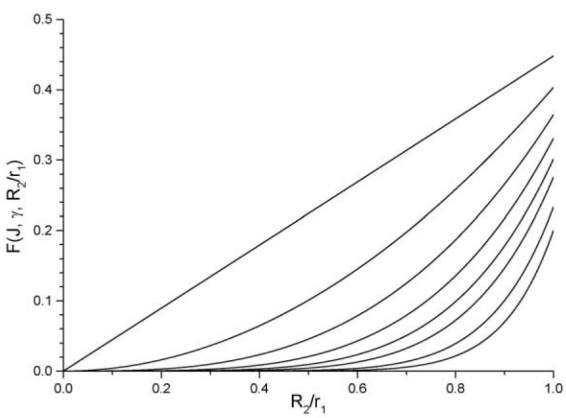

Fig. 4. Function F(J, γ, R_2/r_1) vs. normalized radial coordinate of the current, R_2/r_1, for a coil with $\gamma = 1.25$

3. Effect of random errors in the winding on the position sensitivity

Others have made measurements with different types of Rogowski coils to show that the position sensitivity may be mitigated by reducing random errors in the winding and reducing the gap between the two ends of the coil [3]. They developed techniques to prepare coils with a precision of 50 µm to reduce the position sensitivity to as low as 0.1%, whereas commercial Rogowski coils generally have a position sensitivity greater than 1%. One of their techniques is laser machining of a thin layer of conductive material that is fired onto a core that is made from a machinable ceramic [4]. One coil that was made in this manner has 130 turns with the dimensions r_1 = 5.1 cm and r_2 = 7.6 cm ($\gamma = 1.49$). The criterion for a precision of 50 µm with this coil would correspond to 0.1 % of r_1 for this device.

We have extended the present analysis to develop the following method for determining the effects of random errors in the fabrication on the position sensitivity of a Rogowski coil:

1. N points are specified which are evenly spaced on a circle with radius $r = r_1$, corresponding to the ends of each of the N turns of a uniformly wound coil.
2. Each of the N points is displaced by adding Gaussian-distributed random errors to the angular coordinate θ, so the positions at the ends of each turn are displaced by distances having an rms value that is approximately 0.1% of r_1.
3. This array of points is used to determine local values of the turn density N'(θ) for a nonuniformly wound coil.
4. The Fourier coefficients for Eq. (5) are determined by using the local values of N'(θ) with numerical integration with Eqs. (8A)-(8C).
5. The Fourier coefficients are used in the following equation, derived from Eq. (6), to determine the fractional error in the open-circuit induced voltage when the current is located at each of a large number of points on a ring with radius R_2, and then the rms variation of these voltages is calculated as a measure of the position sensitivity.

$$\frac{\Delta V_{OC}}{V_{OC0}} = \frac{1}{2N_0'\ln(\gamma)} \bullet \sum_{J=1}^{\infty}\left[\frac{\left[N_{JC}'\cos(J\Phi)+N_{JS}'\sin(J\Phi)\right]}{J} \bullet \left(\frac{R_2}{r_1}\right)^J\left[1-\frac{1}{\gamma^J}\right]\right] \qquad (12)$$

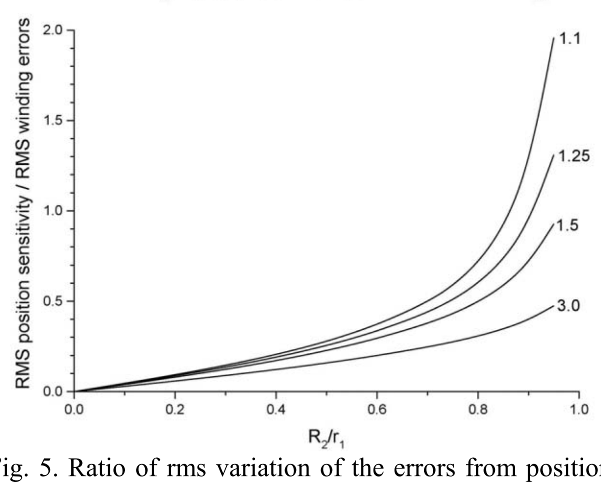

Fig. 5. Ratio of rms variation of the errors from position sensitivity to rms fractional errors in the coil vs. normalized radial coordinate of the current.

Figure 5 shows the effect of random errors in the winding on the position sensitivity. When the errors are small, the variation that is caused by position sensitivity is proportional to the errors in the locations of the points in the winding. Thus, the ordinate in Fig. 5 is the ratio of the rms variation of the errors caused by position sensitivity, to the rms fractional errors in placing the end points of the turns of the coil. This quantity is plotted as a function of the normalized radial coordinate of the current, R_2/r_1, for coils having a relative width, γ = 1.10, 1.25, 1.50, and 3.00.

SUMMARY AND CONCLUSIONS

Figures 2 and 3 in the first example are consistent with numerical solutions [5] and measurements [3,5] by others. These two figures show that the position sensitivity becomes large when the current is in a small wire or particle beam that is far from the axis of the toroid. However, the full magnitude of this effect would not be seen in some practical applications of Rogowski coils such as when the current is in a bus bar that occupies 56% of the cross-sectional area of the aperture of the coil [3]. Manufacturers of Rogowski coils generally suggest that the aperture should not be much larger than the size of the conductor. However, this is not possible in measurements with particle beams so such measurements are subject to position sensitivity.

Others have previously noted that increasing the relative width of the coil reduces the position sensitivity that is caused by a gap in the winding [5], and Eq. (9) in the first example shows that this is the case. Furthermore, Eqs. (11A) and (11B) in the second example show that increasing the relative width of the coil also reduces the position sensitivity which is caused by a sinusoidal perturbation in the winding. Furthermore, Eq. (12) in the third example shows that the position sensitivity which is caused by random errors in the winding is also reduced by increasing the relative width of the coil. Figure 5 in the third example shows that with coils that are wound with a specified degree of precision, when the current is located at $R_2/r_1 = 0.8$, the position sensitivity may be reduced by a factor of 2 by increasing γ ($r_2/_{r1}$) from 1.25 to 3.0.

REFERENCES

[1] W. Rogowski and W. Steinhaus, “Die messung der magnetischen spannung,” Arch. Electrotech, 1 (1912) 141-150.

[2] A.G. Klein, “Demonstration of Ampere’s circuital law using a Rogowski coil,” Am. J. Phys. 43 (1975) 368-370.

[3] A. Andersson, D. Destefan, J.D. Ramboz, S. Weiss and J.M. DeHaan, “Unique EHV current probe for calibration and monitoring,” IEEE/PS Transmission and Distribution Conference, Vol. 1 (2001) 379-384.

[4] J.D. Ramboz, “Machinable Rogowski coil, design, and calibration,” IEEE Trans. Instrum. Meas. 45 (1996) 511-51

[5] L. Ferkovic, D. Ilic and R. Malaric, “Analysis of the mutual inductance of a precise Rogowski coil,” Instrumentation and Measurement Technology Conference, IMTC-2007, No. 7685 on the CD for the Conference Proceedings.

[6] W. Grobner and N. Hofreiter, *Integraltafel Zweiter Teil Bestimmte Integrale*, Vienna, Springer-Verlag, 1966, p. 112.